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Possible influence of the two string events on the hadron formation in a nuclear environment

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Abstract. One of the basic assumptions of the string model is that as a result of a DIS in nucleus a single string arises, which then breaks into hadrons. However the pomeron exchange considered in this work, leads to the production of two strings in the one event. The hadrons produced in these events have smaller formation lengths, than those with the same energy produced in the single string events. As a consequence, they undergo more substantial absorption in the nuclear matter.

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1 Introduction

Production of hadrons in a nuclear environment is a well known tool for investigations of the early stage of hadronization. The basic assumption of the parton model is that the virtual photon interacts as a point-like electromagnetic probe. Being directly connected with the charge of the single parton in the target, the virtual photon knocks it out, transferring it's energy. A color string that is stretched between the knocked out parton and the target remnant then breaks into the final hadrons. (see Fig. 1). In this article we consider an additional mechanism connected with the possibility of a simultaneous production of two strings. This happens due to the complicated nature of the photon,[1] which can interact as a hadron-like system consisting of a quark and an antiquark with a gluonic field of the nucleon (pomeron) and by means of quark–antiquark exchange (reggeons). Diagrams describing the mechanism of the high energy hadron–hadron interaction in Regge theory are shown in Fig. 2. The surfaces of planar and cylindrical diagrams are covered by gluons and quark–antiquark pairs, which do not represented for the sake of simplicity.

One should note, that the reggeon exchange leads to a one string picture as in Fig. 1, whereas pomeron exchange leads to a two string mechanism of hadron production. The main goal of this work is to study the contribution of the two string events in an electro-production on nuclei. For this purpose we investigate the nuclear attenuation (NA), which is the ratio of the differential multiplicity on
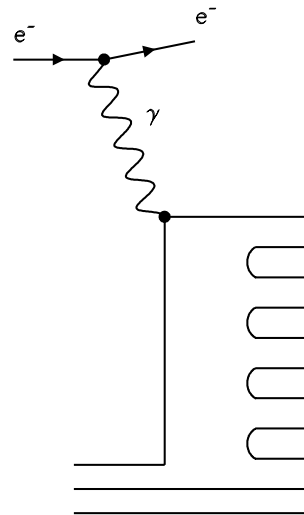
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[1] Photon has properties of an electromagnetic probe and hadron and interacts with a hadrons via three types of processes: direct, VMD, and anomalous.

Fig. 1. The single string mechanism of the hadrons electroproduction. The virtual photon interacts as point-like electromagnetic probe with the parton in target, knocks it out

a nucleus (A) to that on deuterium (D). The experimentally measured observable, NA, is usually considered as a function of three kinematical variables: energy of the virtual photon ν, fraction of photon's energy carried by final hadron $z = E_h/\nu$, where E_h is the energy of hadron in laboratory system and square of the four momentum of photon $q^2 = -Q^2$

$$R_{\mathrm{M}}^h(\nu, z, Q^2) = \frac{\left(\frac{N^h(\nu,z,Q^2)}{N^e(\nu,Q^2)}\right)_A}{\left(\frac{N^h(\nu,z,Q^2)}{N^e(\nu,Q^2)}\right)_D},$$

with $N^h(\nu, z, Q^2)$ being the number of semi-inclusive hadrons in a given (ν, z, Q^2) bin, and $N^e(\nu, Q^2)$ being the number of inclusive DIS leptons in the same (ν, Q^2) bin. However, we consider R_{M}^h as a function of two variables,
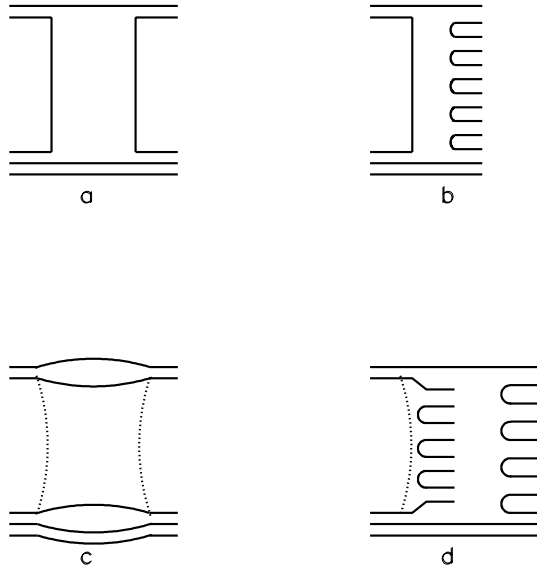
Fig. 2. The hadron–hadron interaction in the Regge theory. The interaction via exchange of reggeon (**a**) and pomeron (**c**) in elastic scattering is shown. Contributions of reggeons and pomeron in a multi-particle production are shown in **b** and **d**

(ν, Q^2) or (z, Q^2), which assumes that integration over the third kinematic variable is done. As it has been shown, the contribution of the two string mechanism in R_{M}^h is negligible at $Q^2 > 10\,\mathrm{GeV}^2$. It is in order of few percent of the basic (single string) mechanism in case of the HERMES kinematics ($Q^2 \approx 2.5\,\mathrm{GeV}^2$). However it can increase essentially at lower values of Q^2.

The paper is organized as follows. In Sect. 2 we briefly discuss how the fraction of two string events can be obtained. Formulae for the calculation of the virtual photon energy division between two strings are presented in Sect. 3. Section 4 presents the results and discussion. Our conclusions are presented in Sect. 5.

2 Fraction of two string events

Total hadronic cross sections show a characteristic fall-off at low energies and a slow rise at higher energies. This behavior can be parametrized by the form [1]

$$\sigma_{\mathrm{tot}}^{AB}(s) = X^{AB} s^\epsilon + Y^{AB} s^{-\eta} \quad [\mathrm{mb}] , \tag{1}$$

for $A + B \to X$ and $s = (p_A + p_B)^2$, with s in GeV^2. The powers ϵ and η are universal, with fit values

$$\epsilon \approx 0.0808 , \quad \eta \approx 0.4525 , \tag{2}$$

while the coefficients X^{AB} and Y^{AB} are process-dependent. Equation (1) can be interpreted within Regge theory, where the first term corresponds to the pomeron exchange and gives the asymptotic rise of the cross section. The second term, the reggeon one, is mainly of interest at low energies. Empirically, the γp data are well described by the

s-dependence of type (1)

$$\sigma_{\mathrm{tot}}^{\gamma p}(s) \approx 67.7 s^\epsilon + 129 s^{-\eta} \quad [\mu\mathrm{b}] . \tag{3}$$

Actually, the above formula is a prediction [1] preceding the HERA data [2, 3]. On the other side, it is well known that the total γp cross section may be written as consisting from three parts (see for instance [4–6])

$$\sigma_{\mathrm{tot}}^{\gamma p} = \sigma_{\mathrm{VMD}}^{\gamma p} + \sigma_{\mathrm{dir}}^{\gamma p} + \sigma_{\mathrm{anom}}^{\gamma p} , \tag{4}$$

where the subdivision of total cross section corresponds to the existence of three main event classes in γp events:

1. The vector meson dominance (VMD) processes, where the photon turns into a vector meson before the interaction with target, and therefore all processes allowed in hadronic physics may occur.
2. The direct processes, where a bare photon interacts with a parton from the proton.
3. The anomalous processes, where the photon perturbatively branches into a $q\overline{q}$ pair, and one of these interacts with a parton from the proton.

The total VMD cross section is obtained as weighted sums of the allowed vector-meson states [4–7]

$$\sigma_{\mathrm{VMD}}^{\gamma p} = \sum_{V = \rho^0, \omega, \phi} \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2} \sigma_{\mathrm{tot}}^{Vp} , \tag{5}$$

where $\alpha_{\mathrm{em}} \approx 1/137$, $f_V^2/4\pi$ determined from data to be 2.20 for ρ^0, 23.6 for ω and 18.4 for ϕ [7], $\sigma_{\mathrm{tot}}^{Vp}$ are corresponding vector meson–proton total cross sections, which can be find, for instance in [4–6].

Now, let us turn to the electro-production process, when one is dealing with the virtual photon interaction. The phenomenological model which extends VMD to the case of the virtual photon is the generalized VMD (GVMD) [8]. In this case we have a more complicated connection with the vector meson total cross sections

$$\sigma_{\mathrm{GVMD}}^{\gamma^* p} = \sum_{V = \rho^0, \omega, \phi} \frac{4\pi\alpha_{\mathrm{em}}}{f_V^2} \frac{1}{\left(1 + Q^2/m_V^2\right)^2} \sigma_{\mathrm{tot}}^{Vp} , \tag{6}$$

where $\sigma_{\mathrm{GVMD}}^{\gamma^* p}$ is function of two variables: the c.m. energy of the $\gamma^* p$ system W and Q^2. The total $\gamma^* p$ cross section, $\sigma_{\mathrm{tot}}^{\gamma^* p}$, can be related to the proton structure function F_2 through the relation (see, for instance, [9, 10])

$$\sigma_{\mathrm{tot}}^{\gamma^* p}(x, Q^2) = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2(1-x)} \frac{Q^2 + 4m_p^2 x^2}{Q^2} F_2(x, Q^2) , \tag{7}$$

where the total $\gamma^* p$ cross section includes both the cross section for the absorption of transverse and of longitudinal photons, x is Bjorken variable $x = Q^2/2m_p\nu$, where m_p is the proton mass. Proton structure function F_2 was calculated using a model presented in [11]. Using parameterization (1) for $\sigma_{\mathrm{tot}}^{Vp}$ for $V = \rho^0, \omega, \phi$ as presented in [4–6], (6) can be written separately for the contribution of pomeron in $\gamma^* p$ total cross section $\sigma_{\mathrm{GVMD}}^{\gamma^* p(P)}$. The knowledge of this quantity allows one to obtain the relative share of the

contribution of the pomeron in the virtual photon–proton total cross section

$$\alpha = \frac{\sigma^{\gamma^* p(P)}_{\text{GVMD}}}{\sigma^{\gamma^* p}_{\text{tot}}} \, . \tag{8}$$

The Regge theory is widely used to describe the low p_T high-energy interactions of hadrons, nuclei, real and virtual photons. The Pomeranchuk singularity plays a special role in this theory as it determines the high energy behaviour of diffractive processes and multi-particle production. It is possible to connect the general results of the Regge theory with the parton model. We will follow the representation of dual parton model (DPM) [12], which was developed by incorporating partonic ideas into dual topological unitarization.[2] In DPM a meson (baryon) are a excitations of an open string with valence quark and antiquark (diquark) at its ends. When a string is stretched, it decays into hadrons by breaking into short strings. The dominant contribution to the high energy scattering of two hadrons comes from a closed string (a pomeron) exchange, having a cylinder topology (see Fig. 2c). A unitarity cut of the cylindrical pomeron shows that the sources of the multi-particle production are two hadronic chains (see Fig. 2d). Other reggeons correspond to the planar diagrams, which give one hadronic chain in the multiparticle production. The presence of contribution from pomeron leads to the arising of the two string events in DIS in addition to the single string events. The relative share of these events given by (8).

3 Energy division between two strings

In the two string events the energy of the projectile (virtual photon) is shared by two strings. We will use two methods for the definition of this energy division.

As a first method, we will use the functions $W_{T,L}(\beta, r_T)$ from [14], which play a role of the square of the wave functions of the virtual photon's quark–antiquark fluctuations. In an explicit form:

$$W_T(\beta, r_T) = \frac{6\alpha_{\text{em}}}{(2\pi)^2} \sum_{i=1}^{N_f} e_i^2 \big\{ [1 - 2\beta(1-\beta)]\varepsilon^2 K_1^2(\varepsilon r_T) \\ + m_i^2 K_0^2(\varepsilon r_T) \big\} \tag{9}$$

and

$$W_L(\beta, r_T) = \frac{6\alpha_{\text{em}}}{(2\pi)^2} \sum_{i=1}^{N_f} e_i^2 4Q^2 \beta^2 (1-\beta)^2 K_0^2(\varepsilon r_T), \tag{10}$$

where m_i is the mass of the quark i, e_i is the quark charge, β is the fraction of the $q\bar{q}$ momentum carried by one of the quarks, $\varepsilon^2 = m_i^2 + \beta(1-\beta)Q^2$, r_T is the transverse size of the $q\bar{q}$ pair, $K_{0,1}(x)$ are modified Bessel functions. It is impossible to normalize the virtual photon wave function to

unity because the normalization integral

$$N_{\gamma^*} = \int_0^1 \mathrm{d}\beta \int \mathrm{d}^2 r_T W_T(\beta, r_T) \tag{11}$$

diverges logarithmically at small distances, since $K_1(x) \sim 1/x$ at $x \to 0$. This divergence does not cause any problems if one includes in the normalization integral the dipole–nucleon cross section, since $\sigma_{q\bar{q}}(r_T) \sim r_T^2$ at small r_T. Then one obtains the probability function for the energy division between two strings normalized to unity in form:

$$w(\beta) = \frac{\int \mathrm{d}^2 r_T \sigma_{q\bar{q}}(r_T)[W_T(\beta, r_T) + \epsilon W_L(\beta, r_T)]}{\int_0^1 \mathrm{d}\beta \int \mathrm{d}^2 r_T \sigma_{q\bar{q}}(r_T)[W_T(\beta, r_T) + \epsilon W_L(\beta, r_T)]}, \tag{12}$$

where ϵ is photon polarization and $\sigma_{q\bar{q}}(r_T)$, as mentioned above, is the cross section for the dipole–nucleon interaction.[3]

As the second method we will use a probability function for the energy division $w(y_q, y_{\bar{q}})$ in form presented in [16, 17]. This function can be applied to any hadronic system that consists of a quark and antiquark

$$w(y_q, y_{\bar{q}}) = C \exp(-(1 - \alpha_R(0))|y_q - y_{\bar{q}}|), \tag{13}$$

where y_q and $y_{\bar{q}}$ are rapidities of the quark and the antiquark, respectively, $\alpha_R(0)$ is an intercept of the secondary Regge pole α_R ($R = \rho, \omega, f$). The commonly used average value for $\alpha_R(0)$ is $\alpha_R(0) \approx 0.5$, C is a normalization factor. For calculations it is convenient to present the function $w(y_q, y_{\bar{q}})$ in form:

$$w(\beta) \approx C \left[\frac{\min(\beta, 1-\beta)}{\max(\beta, 1-\beta)} \right]^{(1 - \alpha_R(0))}. \tag{14}$$

Strictly speaking, the energy division function in form of (13) and (14) is suitable for the region $|y_q - y_{\bar{q}}| \gg 1$, but in this paper, taking into account its qualitative character, we will use this formula for the full region of $y_q - y_{\bar{q}}$. We will discuss such choice later in results and discussion.

4 Results and discussion

The basic formulae for NA was derived in [18] in form of:

$$R_A = 2\pi \int_0^\infty b \, \mathrm{d}b \\ \times \int_{-\infty}^\infty \mathrm{d}x \rho(b, x) \left[1 - \int_x^\infty \mathrm{d}x' \sigma^{\text{str}}(\Delta x) \rho(b, x') \right]^{A-1}, \tag{15}$$

where b – impact parameter, x – longitudinal coordinate of the DIS point, x' – longitudinal coordinate of the string–

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[2] Similar ideas lie in the base of quark gluon string model [13].

[3] For calculations we use $\sigma_{q\bar{q}}(r_T)$ in the form presented in [15] by (9)–(12). The only difference is that for the pion–nucleon total cross section we use a constant value $\sigma^{\pi p}_{\text{tot}} = 25$ mb.

nucleon interaction point, $\sigma^{\mathrm{str}}(\Delta x)$ – the string–nucleon cross section on distance $\Delta x = x' - x$ from DIS point, $\rho(b, x)$ – nuclear density function. The detailed information related to these quantities can be found in [18].

Using quantities defined in the preceding sections and given above (15) for NA, we can now calculate the NA, taking into account the admixture of the two string events. In order to do this, we must change R_A in (15), to $(1-\alpha)R_A + 2\alpha R'_A$, where R_A and R'_A are the absorption functions for the hadron produced in the single string event and in one of the strings of the two string event, respectively. R'_A is integrated over β with energy division functions $w(\beta)$ presented by (12) and (14).

Now, following the formalism of [18] we can write the NA in form:

$$R_{\mathrm{M}}^h(\nu, z, Q^2) = \frac{(1-\alpha)R_A + 2\alpha R'_A}{(1-\alpha)R_{\mathrm{D}} + 2\alpha R'_{\mathrm{D}}}, \qquad (16)$$

where R_{D} and R'_{D} are the absorption functions for deuterium. The NA for pions on helium, neon and krypton nuclei as a functions of ν and z calculated with $w(\beta)$ obtained from virtual photon wave functions (see (12)), are presented in Figs. 3–5. In Figs. 6–8 the same results are obtained with $w(\beta)$ in form (14). On Figs. 3 and 6 the NA
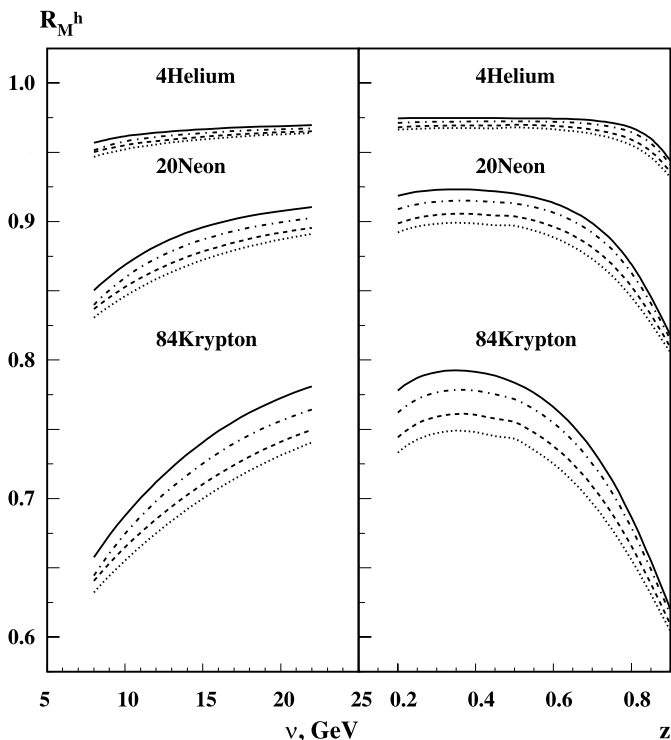

Fig. 4. NA ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of ITSM with τ_c from Lund model. The rest as in caption of Fig. 3
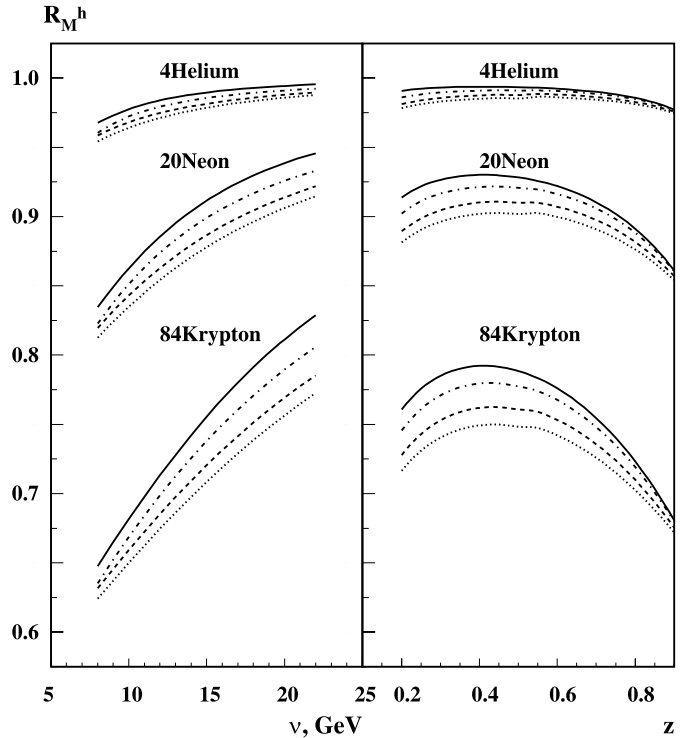

Fig. 3. NA ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of TSM with τ_c from Lund model. *Solid curves* represent single string case; *other curves* represent cases with admixture of two string events. Fraction of two string events depends from the value of Q^2. *Dotted curves* correspond $Q^2 = 1\,\mathrm{GeV}^2$, *dashed* $Q^2 = 2.5\,\mathrm{GeV}^2$ and *dashed-dotted* $Q^2 = 10\,\mathrm{GeV}^2$. In calculation was used function $w(\beta)$ from (12)
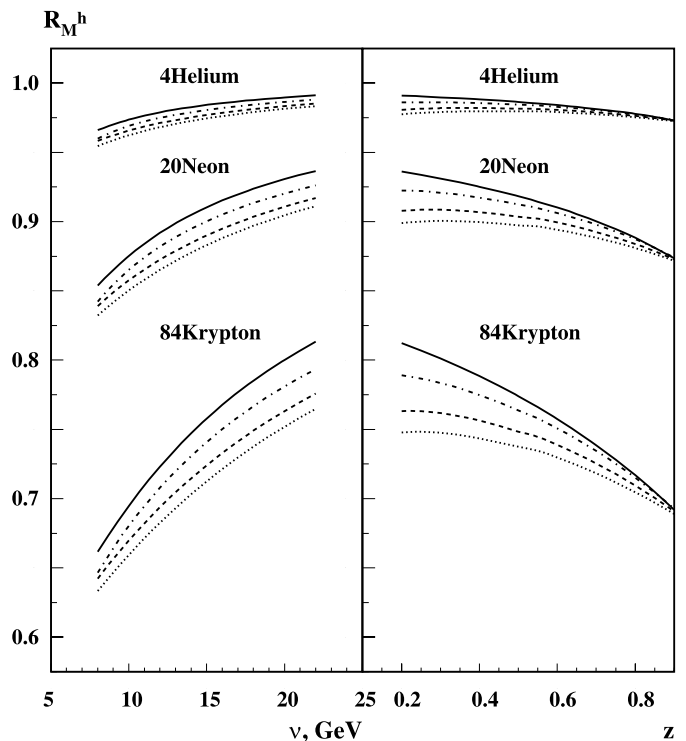

Fig. 5. NA ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of ITSM with τ_c for leading hadron (see [18]). The rest as in caption of Fig. 3
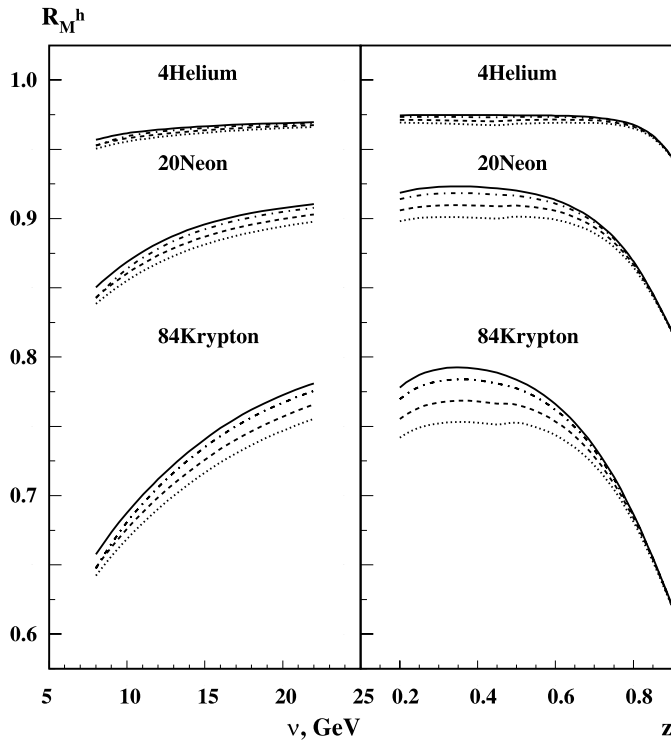
Fig. 6. NA ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of TSM with τ_c from Lund model. *Solid curves* represent single string case; *other curves* represent cases with admixture of two string events. Fraction of two string events depends from the value of Q^2. *Dotted curves* correspond $Q^2 = 1\,\mathrm{GeV}^2$, *dashed* $Q^2 = 2.5\,\mathrm{GeV}^2$ and *dashed-dotted* $Q^2 = 10\,\mathrm{GeV}^2$. In calculation was used function $w(\beta)$ from (14)

Fig. 7. NA ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of ITSM with τ_c from Lund model. The rest as in caption of Fig. 6

in the framework of TSM with $\tau_c(4)$ (Lund model)[4] are used. Solid curves represent single string case, while other curves represent cases with admixture of the two string events. The fraction of the two string events depends from the value of Q^2. Dotted curves correspond $Q^2 = 1\,\mathrm{GeV}^2$, dashed $Q^2 = 2.5\,\mathrm{GeV}^2$ and dashed-dotted $Q^2 = 10\,\mathrm{GeV}^2$. In Figs. 4 and 7 the NA in the framework of the ITSM with $\tau_c(4)$ are presented. Figures 5 and 8 show the NA in the framework of ITSM with $\tau_c(3)$ (leading hadron). Let us now discuss the results of calculations. First of all one should note several common features of the results: a) admixture of two string events always increases the effect of nuclear attenuation, i.e. increases gap between unity and value of R_{M}^h; b) contribution of the two string events increases with the increase of the atomic mass number; c) contribution of the two string events also increases with larger values of ν, and decreases with the larger values of z; d) z-dependence is more sensitive to the contribution of the two string events than the ν-dependence. From Figs. 3–8 one can see that the contribution of the two string events is negligible for all nuclei at $Q^2 > 10\,\mathrm{GeV}^2$. At mod-

Fig. 8. Nuclear attenuation ratio for pions on different nuclei as a functions of ν (*left panel*) and z (*right panel*) in the framework of ITSM with τ_c for leading hadron (see [18]). The rest as in caption of Fig. 6

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[4] In this paper the models and the constituent formation times τ_c are marked as in [18]. In particular, $\tau_c(3)$ and $\tau_c(4)$ means that τ_c is taken in form (3) and (4) of the referred paper.
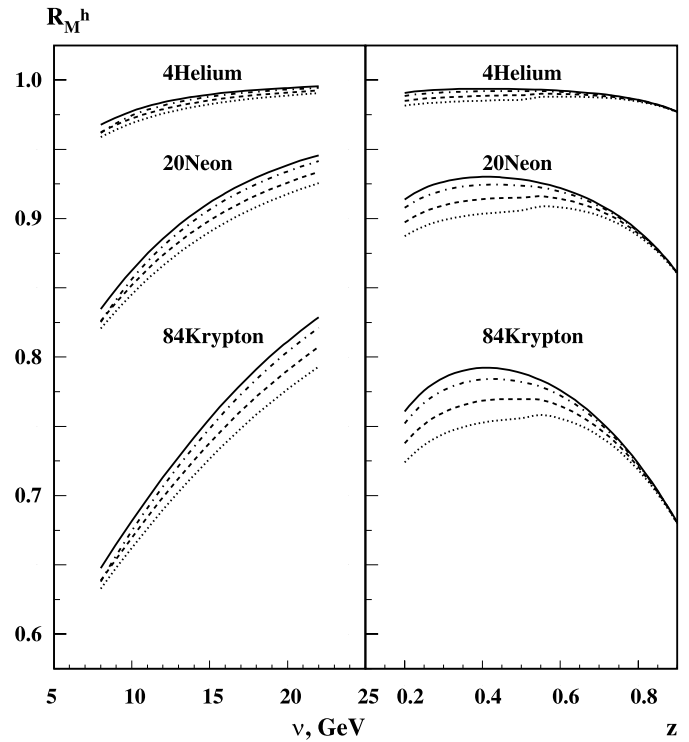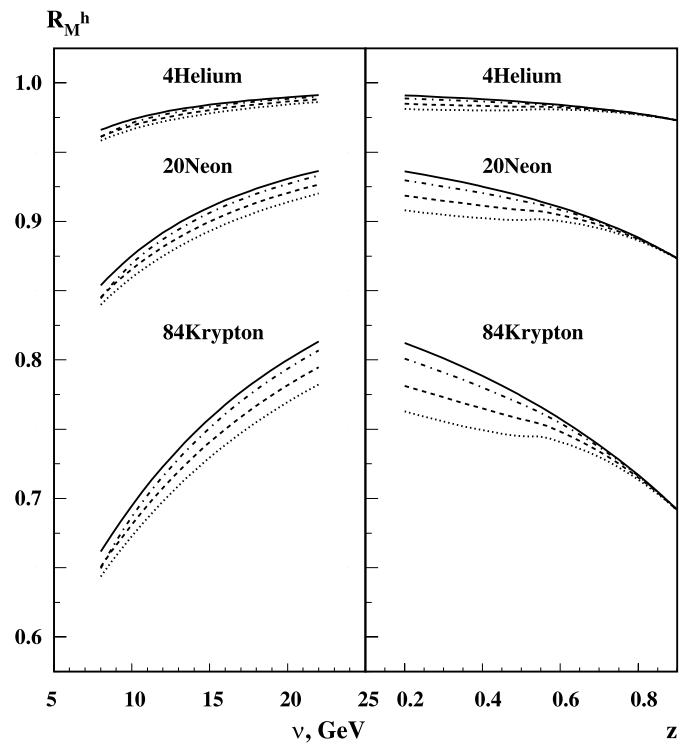
erate Q^2 it becomes more essential. For instance at $Q^2 = 2.5\,\mathrm{GeV}^2$, which corresponds to the HERMES kinematics, the contribution of the two string events in z-dependence on krypton achieves 3%–4%. For $Q^2 = 1\,\mathrm{GeV}^2$ it increases to 5%–6% (As an observable we use the NA ratio R_{M}^h. If nuclear effects are absent, it is obviously equal to unity. The quantity directly connected with the nuclear effects is $1 - R_{\mathrm{M}}^h$. The relative contribution of the two string events in this quantity is several times larger than in R_{M}^h.). Figures 3–8 also show that the contribution of the two string events is always small for helium. Comparing Figs. 3–5 with Figs. 6–8 one can see that the calculations with functions $w(\beta)$ defined in (12) and (14) give results that are close in shape, however the contribution of two string events calculated with $w(\beta)$ as in (12) is more significant numerically. We have performed an additional test for the probability function of the energy division $w(y_q, y_{\bar{q}})$. Besides (14) we also used an expression $w(y_q, y_{\bar{q}}) = \mathrm{const}$; and also a combined expression when constant value of w was used for close values of y_q and $y_{\bar{q}}$ and (14) was used for the case $|y_q - y_{\bar{q}}| \gg 1$. Changing the expressions for w leads to slight change of the form of the curves, but does not change the contribution level significantly. In this paper we have limited ourself to consideration of NA for pions only, but the described mechanism of the two string admixture is common for all mesons and baryons, and extension is quite straightforward. We have studied the two string admixture in the framework of the string model, but in any model that deals with the NA there is a need to take into account the fact that the virtual photon can transfer its energy both single parton and to two partons. The last point which we would like to discuss is what happens if, instead of a single string fit of the NA data (performed in [18]), we perform a fit including two string admixture to the single string mechanism. We do not expect that the numerical values of the fitting parameters and χ^2 will change significantly. Fitting parameters σ_q and σ_s in TSM and σ_q and c in ITSM, should become slightly smaller (in order of $\sim 10\%$), and all curves should have a somewhat smoother behavior (definitions of free parameters see in [18]).

5 Conclusions

- For the first time the contribution of two string events in the NA was investigated. Admixture of the two string events always increases the effect of NA, i.e. increases gap between unity and R_{M}^h value.
- Contribution of two string events increases with the increasing of the atomic mass number; as well as with the increasing of ν, and decreases with the increasing of z. It is more prominent in the z-dependence.
- At moderate Q^2 contribution of the two string events is essential enough. For instance in the z-dependence on krypton it is approximately 5%–6% at $Q^2 = 1\,\mathrm{GeV}^2$, 3%–4% at $Q^2 = 2.5\,\mathrm{GeV}^2$. It is negligible for all nuclei at $Q^2 > 10\,\mathrm{GeV}^2$.
- It is very interesting to investigate the contribution of two string events in case of double hadron production [19, 20].

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